

# Entropy in the Present and Early Universe

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## Abstract

This is a short analysis of the changes in the concept of entropy as applied to physics of the present-day and Early Universe. Of special interest is a leading role of such a notion as deformation of a physical theory. The relation to a symmetry of the corresponding theory is noted. As this work is not a survey, the relevant author's works are mainly considered. This paper is to be published in special issue "Symmetry and Entropy" of journal **SYMMETRY: Culture and Science**

**Keywords:** *deformation, entropy, symmetry.*

## 1 Introduction

Two principal theories which are revolutionary for the fundamental physics – quantum mechanics and relativity theory - were created in the XX-th century. The third theory including statistical mechanics and thermodynamics has been formed as far back as the XIX-th century. By the eighties of the last century all these theories have acquired their complete form. They were effectively used, actually enabling solution of all the known problems within the scope of the existing paradigm. However, energies are constantly increasing with the development of the physical experiment. At the present time CERN is testing the operation a Large Hadronic Collider (LHC) that could significantly widen the opportunities for detection of new particles and previously unknown phenomena. Besides, in the last decade a new unique field of theoretical physics - cosmological microphysics - has been formed just at the junction of elementary particle physics and cosmology and, due to the advances in astrophysics and modern technology (space probes, Hubble telescope, etc.), has regained the experimental

status. In the end, all these makes it possible to tackle a number of problems associated with the early Universe (instants immediately after the Big Bang).

Therefore, of critical importance is an extension of the available fundamental physics paradigm to find the answers for the questions concerning the early Universe as all the above-mentioned theories work well on the scale of the energies known in the every-day life or in modern accelerators, failing with the energies of the Big Bang. Specifically, the general relativity theory in this case seems to lose its force of prediction: it is unknown what happens with the space-time at Planck's scales ( $10^{-33}cm$ ), most likely ceasing to be continuous. A quantum theory should be modified in a similar way, in particular, due to the divergence problems preventing the correct results in the region of higher energies. A new paradigm should extend the old one rather than ruin it. Any theory in physics has its applicability limits. For all the basic theories these limits are associated with some energy limits. Because of this, the paradigm should expand the limits so that in the low-energy limit the well-known theories could accurately provide the result experimentally checked over and over.

**With my paper I would like to pay a tribute to my first Since Manager Professor Alexandre Zaleski who will be celebrating his 70-th anniversary in January 2009.**

## 2 “Deformed” physical theories

As indicated above, any new paradigm is a change in the old one to extend its applicability limits. This is true not only in physics. In every field of science or culture, a new paradigm arises when considering the problem that, as a rule, necessitates widening of the initial scope for the successful solution. Such a situation is typical for the development of a new method, concept or idea.

In physics a new paradigm is usually related to new constants and variables. To illustrate, the “advent” of a new constant (Planck's constant  $\hbar$ ) in quantum mechanics as compared to the classical one was followed by the development of a completely new mathematical apparatus. It can be said that the classical mechanics appears from the quantum mechanics because of passage to the limit  $\hbar \rightarrow 0$ .

There is a similar passage from nonrelativistic to relativistic dynamics **Faddeev (1989, p.15)** associated with a change in the movement group

of space-time: from Galilean transformations to those of Lorentz-Poincaré. Both these groups are ten-parametric.

Galilean transformation is as follows

$$\vec{x} \rightarrow \vec{x} + \vec{v}t, \quad t \rightarrow t \quad , \quad (1)$$

and in the case of Lorentz transformations we have

$$\vec{x} \rightarrow \frac{\vec{x} + \vec{v}t}{\sqrt{1 - v^2/c^2}} \quad (2)$$

$$t \rightarrow \frac{t + (\vec{v} \cdot \vec{x})/c^2}{\sqrt{1 - v^2/c^2}} \quad , \quad (3)$$

where  $\vec{x}$  are the coordinates,  $t$  is the time,  $\vec{v}$  is the relative velocity of the reference system in motion.

As compared to Galilean transformations, those of Lorentz involve the fixed constant  $c$  - speed of light - that is equal to  $3 \cdot 10^{10} cm/s$ . When  $c \rightarrow \infty$ , expressions in (2) change over to (1), i.e. Lorentzian transformations become those of Galileo (and similarly, the corresponding groups and their Lie algebras).

Now we introduce the definition of a “deformed” physical theory.

**Definition 1.** Deformation of a physical theory is understood as its extension due to the introduction of one or several additional parameters so that the initial theory appears on passage to the limit.

In terms of Definition 1, the relativistic dynamics group represents deformation of the group for nonrelativistic motion, where the deformation parameter is  $1/c^2$  **Faddeev (1989, p.15)**.

In a similar way, quantum mechanics represents “deformation” of the classical one with the deformation parameter  $\hbar$  (or  $\frac{1}{\hbar}$ ). In the following section this example will be treated in greater detail. Now, using **Faddeev’s** approach, we consider such a notion as stable deformation: **Faddeev (1989, p.15)** “... In a mathematical theory of the algebraic structure deformations there is a notion of the stable structure. It is said that a structure is stable when all similar deformations are equivalent to this structure.”. “From this viewpoint, quantum mechanics is stable as opposed to the classical mechanics allowing for the nonequivalent deformation – quantum mechanics”. Here by “similar” we mean the structures with a deformation parameter close in its value to the initial one **Gerstenhaber (1964)**.

**Definition 2.** Stable deformation of a physical theory is recognized as the deformation when all the like (i.e. having similar deformation parameters) deformations are equivalent.

Despite the fact that in quantum mechanics  $\hbar$  is a constant equal to  $\simeq 10^{-27} \text{gcm}^2\text{s}^{-1}$  one can easily imagine “quite a number” of quantum mechanics with the deformation parameters close to  $\hbar$  but still nonzero, which are equivalent mathematically.

The third principal achievement of physics in the XX-th century is the general relativity theory – stable deformation in terms of Definition 2 meaning that **Faddeev (1989, p.16)**. “a gravitational theory is just based on the replacement of the flat Minkowski space-time by the common-position curved pseudo-Riemann space. It can be argued that in a set of such spaces the flat space is a degeneracy, whereas spaces in the neighborhood of the curved one are also curvilinear. A measure of deformation is determined by the gravitation constant  $G$ , inherited from Newton and introduced into Hilbert-Einstein equation. The dimensionality of this constant is functionally independent of  $\hbar$  and  $c$ , and together with them form the basis for all dimensional parameters.”

### 3 “Deformed” quantum mechanics of the Early Universe. Different approaches.

As noted in the previous section, contrary to the classical mechanics, quantum mechanics involves an additional parameter ( $\hbar$ ) in the region of well-known energies. But at the energies close to those of the Big Bang this hardly is so. An important role of Planck’s quantities in this case is revealed. The Planck’s length is given by

$$\ell_p \equiv \sqrt{\frac{G\hbar}{c^3}} \simeq 1.6 \cdot 10^{-35} m = 1.6 \cdot 10^{-33} cm, \quad (4)$$

the Planck’s time is found from

$$t_p \equiv \frac{\ell_p}{c} = \sqrt{\frac{G\hbar}{c^5}} \simeq 0.54 \cdot 10^{-43} s, \quad (5)$$

the Planck’s mass is obtained as

$$M_p \equiv \frac{\hbar}{c\ell_p} = \sqrt{\frac{\hbar c}{G}} \simeq 2.2 \cdot 10^{-8} kg, \quad (6)$$

and the Planck's energy is determined as

$$E_p = \sqrt{\frac{\hbar c^5}{G}} \simeq 1.2 \cdot 10^{19} GeV. \quad (7)$$

Planck's quantities are understood as the known lengths, times and the like represented by the fundamental constants only. As follows from (4) – (7), Planck's quantities are defined by three fundamental constants ( $\hbar$ ,  $c$ , and  $G$ ) rather than one.

In this way a quantum theory (mechanics) of the early Universe should involve all these fundamental constants  $\hbar$ ,  $c$ ,  $G$  as the principal energies associated with the processes proceeding in the early Universe are comparable to the Planck energy  $E_p$ .

Next we show how the well-known quantum mechanics should be deformed to involve the above-mentioned three constants.

### 3.1 Heisenberg uncertainty relations, generalized uncertainty principle, and deformation of Heisenberg's algebra.

From the classical mechanics it is known that for any particle one can measure an exact value of its coordinate  $x$  and momentum  $p$ .

But in quantum mechanics this is not the case any more. As demonstrated by **Heisenberg (1927)**, the accuracy has a natural limit: the greater accuracy of the coordinate measurements we have the less accurate momentum measurements we get, and vice versa

$$\Delta x \cdot \Delta p \geq \hbar \quad , \quad (8)$$

or equivalently

$$\Delta x \geq \frac{\hbar}{\Delta p}.$$

Expression (8) represents one form of the commonly known uncertainty relations (or uncertainty principle) of Heisenberg who in the process of their derivation has assumed that elementary particles take part only in electromagnetic interactions. This assumption stands to reason since at the known energies the electromagnetic interactions are higher than the gravitational ones by several orders.

Note that in literature (8) is commonly used in the form

$$\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar.$$

But in the early Universe, at Planck's scales, gravitational interaction would be comparable to the electromagnetic ones and should be included. Because of this, relation (8) must be modified so that it can occur in the low-energy limit. According to some modern theories (e.g., string theory) **Kaku (1988)**, relations (8) at the Planck's energies ( $\sim E_p$ ) should be modified as follows **Veneziano (1986)**, **Witten (1996)**

$$\Delta x \geq \frac{\hbar}{\Delta p} + \gamma \ell_p^2 \frac{\Delta p}{\hbar}, \quad (9)$$

where  $\gamma$  is a certain dimensionless coefficient. This modification is referred to as the Generalized Uncertainty Principle (GUP)

Inequality (9) has been derived beyond the string theory as well **Adler, Santiago, (1999)** .

On change-over to low energies, the second term in the right side of (9) becomes negligible modifying relation (9) to (8). At the same time, one should remember that, in contrast to the classical mechanics, coordinates and momenta in quantum mechanics are not numbers but operators, mathematically represented by matrices (infinite in the general case) generating the Heisenberg algebra **Dirac (1958)**, **Messiah (1965)** and given by the commutation relations

$$[q_i, q_j] = 0, \quad [p_j, p_k] = 0, \quad [q_j, p_k] = i\hbar\delta_{jk} \quad (10)$$

As usual, in formula (10)  $q_i$  is the operator of the  $i$ -th coordinate,  $p_j - j$ -th momentum,  $\delta_{jk}$  is the Kronecker delta, and  $[a, b] = ab - ba$ .

Consequently, deformation of quantum mechanics in the Early Universe should be Heisenberg's algebra deformation but including all three fundamental constants.

All these deformations in the low-energy limit should lead to commutation relations (10). Nonequivalent deformations may be numerous in accordance with various sequences converging to one and the same limit **Kempf, Mangano, Mann (1995)**, **Maggiore (1993)**.

To illustrate, two numerical sequences with a common term  $a_n = 1/n$  and  $b_n = (-1)^n/n^2$  at  $n \rightarrow \infty$  have the same limit 0, still being absolutely

different. Also, it should be noted that a common feature of all the deformations mentioned is the noncommutativity (or nonpermutable nature) of the operators with coordinates  $q_i$  and  $q_j$  for different  $i$  and  $j$ . This means that for the high-energy deformation of Heisenberg algebra we have

$$[q_i, q_j] \neq 0. \quad (11)$$

Relation (10),  $[q_i, q_j] = 0$ , appears on going to low energies, therefore in the low-energy limit, where the quantum gravitational effects are negligible, noncommutativity (11) is of no importance.

### 3.2 Density matrix deformation at Planck's scales

There is an alternative approach to the deformation of quantum mechanics in the early Universe. The approach is associated with the density matrix deformation and not with the deformation of Heisenberg algebra **Blum (1981)**. The density matrix  $\rho$  is the statistical operator that may be used to calculate the average for any physical quantity in quantum mechanics. This operator was introduced by J. Neuman and L.D. Landau in 1927.

Let us consider inequalities (8) and (9) more closely. The distinguishing feature of the first inequality is its linearity, i.e. all the involved quantities are of the first order, whereas the second inequality is quadratic involving the squares. From the viewpoint of mathematics, in the case under consideration there exists a “minimum length”

$$\Delta x \geq \ell_{\min} \sim \ell_p, \quad (12)$$

and every coordinate measurement may be performed to an accuracy that is not in excess of a particular minimum length  $\ell_{\min}$  of the order of Planck's length. Also, this suggests that there is a maximum energy on the order of Planck's energy  $E_{\max} \sim E_p$ .

Thus, a distinctive feature of quantum mechanics of the Early Universe is the existence of a minimum length,  $\ell_{\min} \sim \ell_p$ , that is referred to as the fundamental length too.

For our further consideration the existence of a minimum length is the starting point.

Since in quantum mechanics the measuring procedure is determined by the density matrix, the problem is as follows. Provided quantum mechanics involves the fundamental length, the question is: “How does the

density matrix on the retention of the measuring procedure change?" In this case the measuring procedure represents an algorithm used to calculate the averages of operators. As demonstrated in previous works (**Shalyt-Margolin and Suarez 2003, Chap. 3**), (**Shalyt-Margolin 2005, Chap. 3**), one of the approaches to this problem is associated with the density matrix deformation  $\rho$ . In so doing  $\rho$  becomes dependent on a new small dimensionless parameter  $\alpha = \ell_{\min}^2/\ell^2$ , where  $\ell_{\min}$  - minimum length, and  $\ell$  - measuring scale determined by the energy measured. The parameter  $\alpha$  may be represented in terms of energy  $\alpha = E^2/E_{\max}^2$ , where  $E$  - measured energy,  $E_{\max} \sim E_p$  - energy of the process. This new parameter is measured over the interval  $0 < \alpha \leq 1/4$ . At low energies  $\rho(\alpha)$  becomes the density matrix of the well-known quantum mechanics  $\rho$

$$\lim_{\alpha \rightarrow 0} \rho(\alpha) = \rho. \quad (13)$$

Note that the parameter  $\alpha$  includes all the above-mentioned fundamental constants ( $\hbar$ ,  $c$ , and  $G$ ) because  $\ell_{\min}$  and  $E_{\max}$  are expressed in terms of these constants.

Heisenberg algebra is subjected to deformation too. As this takes place in the simplest and minimal variant, the first and second relations in (10) remain invariable, whereas the last one is changed as follows:

$$[q_j, p_k] = i\hbar\lambda(\alpha)\delta_{jk}, \quad (14)$$

where  $\lambda(\alpha)$  - function of  $\alpha$  characterized by the property

$$\lim_{\alpha \rightarrow 0} \lambda(\alpha) = 1. \quad (15)$$

The function  $\lambda(\alpha) = \exp(-\alpha)$ , in particular, meets this property.

So, we actually get deformation of Heisenberg algebra that changes to the conventional Heisenberg algebra at low energies.

But to derive the generalized uncertainty relations (9), the foregoing "minimal" variant of the Heisenberg algebra deformation is insufficient - an extended variant is required, where  $[q_i, q_j] \neq 0$  as in formula (11). What form takes this "non-minimal" deformation variant of Heisenberg algebra is presently unknown and remains to be found.



## 4 Deformed statistical theory of the Early Universe

As noted in the preceding section, the uncertainty relations may be modified (deformed) in the quantum mechanics of the early Universe.

Recall that the pair  $(p, x)$  in formulae (8), (9) is called the conjugate pair of variables in quantum mechanics **Messiah (1965, Chap. 4, Sect 2)**. Besides, there is another conjugate pair  $(E, t)$  and hence in terms of this pair (8) has an analog **Messiah (1965, Chap. 4, Sect 2)**

$$\Delta E \Delta t \geq \hbar. \quad (16)$$

Relation (16) relates the uncertainty  $\Delta E$  of the value assumed by this dynamic variable to the time interval  $\Delta t$  characteristic for the time evolution of a system.

Relation (16) possesses an explicit physical meaning: the energy measurement accuracy  $\Delta E$  is related to the time  $\Delta t$  required for this measurement. In particular, (16) indicates that the lower the time interval the poorer the measuring accuracy. However, relations (16) and (8) are distinguished by one more fundamental feature. In (8) both conjugate variables  $p$  and  $x$  are operators of quantum mechanics, whereas in (16) only energy  $E$  is such an operator. Both (16) and (8) are modified at Planck's scales (in the early Universe) **Shalyt-Margolin and Tregubovich (2004, p.73)**, **Shalyt-Margolin (2005, p.62)**

$$\Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_p^2 \frac{\Delta E}{\hbar}. \quad (17)$$

And both modified relations (9) and (17) may be written in the canonical form as

$$\begin{cases} \Delta x \geq \frac{\hbar}{\Delta p} + \gamma \left( \frac{\Delta p}{P_{pl}} \right) \frac{\hbar}{P_{pl}} \\ \Delta t \geq \frac{\hbar}{\Delta E} + \gamma \left( \frac{\Delta E}{E_p} \right) \frac{\hbar}{E_p} \end{cases}, \quad (18)$$

where  $P_{pl} = E_p/c = \sqrt{\hbar c^3/G}$  - Planck's momentum.

Now we consider the thermodynamic uncertainty relation between the inverse temperature and interior energy of a macroscopic ensemble **Lavenda (1991, Chap.4, Sect.4.9)**

$$\Delta \frac{1}{T} \geq \frac{k}{\Delta U} \quad (19)$$

where  $k$  is the Boltzmann constant.

**Bohr (1932)** and **Heisenberg (1969)** were the first to point out that such kind of uncertainty principle should take place in thermodynamics.

The uncertainty in thermodynamic measurements occurs as follows **Lavenda (1991, Chap.4, Sect.4.9)**: “Let us think of the simplest case when a system is brought in contact with a thermal reservoir. Energy ceases to be a thermodynamic function, becoming a random variable instead, as the value of all the external parameters determining this system is insufficient to find the energy. And energy of the system fluctuates. Observations of the energy may be used for the estimation of its conjugate intensive quantity. As the thermostat size is decreased, the energy measurements become significantly more accurate, and a certain energy value appears in the limit. On the other hand, measurements of energy become less accurate with a growing thermostat size. Therefore, a particular temperature of a system may be determined in the limit of an infinite thermostat. An infinite thermal reservoir means an infinite heat capacity. As the heat capacity is inversely proportional to the dispersion of thermal fluctuations, in the limit it goes to zero.”

Since the thermodynamic uncertainty relation (19) have been proved by many authors and in various ways (**Lavenda 1991, Chap.4; Uffink and van Lith-van Dis 1999 and references in them**), their validity is unquestionable. Nevertheless, relation (19) was established using a standard model for the infinite-capacity heat bath encompassing the ensemble. But it is obvious from the above inequalities that at very high energies the capacity of the heat bath can no longer be assumed infinite at the Planck scale. Indeed, the total energy of the pair heat bath - ensemble may be arbitrary large but finite, merely as the Universe is born at a finite energy. Thus, the quantity that can be interpreted as a temperature of the ensemble must have the upper limit and so does its main quadratic deviation. In other words, the quantity  $\Delta(1/T)$  must be bounded from below. But in this case an additional term should be introduced into (19) **Shalyt-Margolin and Tregubovich (2004, p.74), Shalyt-Margolin (2005, p.68)**

$$\Delta \frac{1}{T} \geq \frac{k}{\Delta U} + \eta \Delta U \quad (20)$$

where  $\eta$  is a coefficient. Dimension and symmetry reasons give

$$\eta \sim \frac{k}{E_p^2}.$$

Similar to the previous cases, inequality (20) leads to the fundamental (inverse) temperature.

$$T_{max} = \frac{\hbar}{\Delta t_{min} k}, \quad \beta_{min} = \frac{1}{k T_{max}} = \frac{\Delta t_{min}}{\hbar}. \quad (21)$$

Thus, we obtain the following system of generalized uncertainty relations in the symmetric form

$$\left\{ \begin{array}{l} \Delta x \geq \frac{\hbar}{\Delta p} + \gamma \left( \frac{\Delta p}{P_{pl}} \right) \frac{\hbar}{P_{pl}} + \dots \\ \Delta t \geq \frac{\hbar}{\Delta E} + \gamma \left( \frac{\Delta E}{E_p} \right) \frac{\hbar}{E_p} + \dots \\ \Delta \frac{1}{T} \geq \frac{k}{\Delta U} + \gamma \left( \frac{\Delta U}{E_p} \right) \frac{k}{E_p} + \dots \end{array} \right. \quad (22)$$

or in the equivalent form

$$\left\{ \begin{array}{l} \Delta x \geq \frac{\hbar}{\Delta p} + \gamma l_p^2 \frac{\Delta p}{\hbar} + \dots \\ \Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_p^2 \frac{\Delta E}{\hbar} + \dots \\ \Delta \frac{1}{T} \geq \frac{k}{\Delta U} + \gamma \frac{1}{T_p^2} \frac{\Delta U}{k} + \dots \end{array} \right. \quad (23)$$

Here  $T_p$  is the Planck temperature  $T_p = E_p/k$ , dots in the right side of (22) and (23) denote terms of the higher-order smallness. Also, it is assumed that the factor  $\eta$  in the right side of (20) is equal to  $\gamma k/E_p^2 = \gamma/kT_p^2$ . Note that the last-mentioned inequality is symmetrical to the second one with respect to substitution

$$t \mapsto \frac{1}{T}, \hbar \mapsto k, \Delta E \mapsto \Delta U.$$

However, this observation can by no means be regarded as a rigorous proof of the generalized uncertainty relation in thermodynamics.

There is reason to believe that rigorous justification for the latter (thermodynamic) inequalities in systems (22) and (23) may be made by means of a certain deformation of Gibbs distribution.

The generalized uncertainty relations in thermodynamics (22) substantiated by the author **Shalyt-Margolin and Tregubovich (2004, p.74)**,

**Shalyt-Margolin (2005, p.68)** still require a strict mathematical proof, but by the present time the result has been cited in the monograph **Carroll (2006)**.

In the previous work **Shalyt-Margolin and Tregubovich (2004, pp.78–80)**, **Shalyt-Margolin (2005, pp.62–67)** the density matrix deformation method has been developed as applied to statistical mechanics at Planck's temperature. Specifically, it has been proved that statistical mechanics of the early Universe is also modified (deformed) as compared to the well-known statistical mechanics. In this modified statistical mechanics, in particular, any ensemble has a temperature that could not be higher than some maximum temperature  $T_{\max} \sim T_p$ . Because of this, the statistical density matrix for very high temperatures is deformed too. It begins to be dependent on the new dimensionless parameters  $\tau = T^2/T_{\max}^2$ , where  $T$  - ensemble temperature

$$\rho_{stat} \rightarrow \rho_{stat}(\tau)$$

$$\lim_{\tau \rightarrow 0} \rho_{stat}(\tau) = \rho_{stat}.$$

Similar to the parameter  $\alpha$ , the parameter  $\tau$  is varying over the interval  $0 < \tau \leq 1/4$ . At low temperatures, passage to the limit is also valid with the density matrix  $\rho_{stat}$  for the canonical Gibbs distribution in right hand side of the last formula.

Thus, compared to the well-known statistical mechanics, statistical mechanics of the early Universe (i.e. at super high temperatures of the order of the Planck's temperature) involves a new small parameter, varying over the same interval as the corresponding new small parameter introduced in quantum mechanics at Planck's scales. In this case the statistical density matrix is also deformed so that, in the limit of low temperatures, the statistical density matrix appears corresponding to the canonical Gibbs distribution.

## 5 Entropy in the present and early Universe

Because quantum and statistical mechanics of the early Universe are modified (deformed) compared to the conventional ones at the known energies, the notions involved are also modified, specifically, being dependent on new parameters.

This is true for the entropy notion as well. In this case we use the standard understanding of entropy from the information theory (e.g., see **Adami (2004) pp. 1-3**):

“The concepts of entropy and information quantify the ability of observers to make *predictions*, in particular how well an observer equipped with a specific measurement apparatus can make predictions about another physical system. Shannon entropy (also known as *uncertainty*) is defined for mathematical objects called *random variables*. A random discrete variable  $X$  is a variable that can take on a finite number of discrete states  $x_i$ , where  $i = 1, \dots, N$  with probabilities  $p_i$ . Now, physical systems are not mathematical objects, nor are their states necessarily discrete. However, if we want to quantify our uncertainty about the state of a physical system, then in reality we need to quantify our uncertainty about the *possible outcomes of a measurement of that system*. Our maximal uncertainty about a system is not a property of the system, but rather a joint property of the measurement device and the device with which we are about to examine the system. If our measurement device, for example, is simply a “presence-detector” then the maximal uncertainty we have about the physical system under consideration is 1 bit, which is the amount of *potential information* we can obtain about that system. Thus, the entropy of a physical system is undefined if we do not specify the device that we are going to use to reduce that entropy. Here we consider only the *discrete* version of the Shannon entropy, which is given in terms of the probabilities  $p_i$  as (**Shannon (1948)**):

$$H(X) = - \sum_{i=1}^N p_i \log p_i. \quad (24)$$

For any physical system, how are those probabilities obtained? In principle, this can be done both by experiment and by theory. Once we have defined the  $N$  possible states of the system by choosing a detector for it, the *a priori* maximal entropy, corresponding to the uniform distribution (all states equally likely) is then

$$H_{\max} = \log N. \quad (25)$$

Classical experiments using detector can now sharpen our knowledge of the system. By tabulating the frequency with which each of the  $N$  states appears, we can estimate the probabilities  $p_i$ . Note, however, that this

leads to a biased estimate of the entropy (24), that approaches its true value only in the limit of an infinite number of trials. On the other hand, some of the possible states of the system (or more precisely, possible states of the detector interacting with the system) can be eliminated by using some knowledge of the physics of the system. For example, we may have some initial data about the system. This becomes clear in particular if the degrees of freedom that we choose to characterize the system with are position, momentum, and energy, i.e., if we consider the *thermodynamical entropy* of the system” **Feynman (1972, Chap. 1)**.

The notion of entropy for the quantum system  $A$  was generalized by von Neumann in 1932 as follows:

$$S(\rho_A) = -\text{Tr} \rho_A \log \rho_A, \quad (26)$$

where  $\rho_A$  - density matrix of the system  $A$ , and  $\log \rho_A$  is understood as such an operator that  $e^{\log \rho_A} = \rho_A$ .

But as indicated in the preceding section, new parameters are introduced into a quantum theory of the early Universe. Specifically, in Section 3 it is demonstrated that the deformation of the density matrix is associated with the occurrence of the new parameter  $\alpha$ .

Note that formula (25) may be extended as  $\alpha$  may be included twice **Shalyt-Margolin (2004, 1, p.397, 2004, 2, p.2040)**

$$S_{\alpha_2}^{\alpha_1} = -\text{Tr} \rho(\alpha_1) \log \rho(\alpha_2) \quad (27)$$

where  $0 < \alpha_1, \alpha_2 \leq 1/4$ .

Physically,  $S_{\alpha_2}^{\alpha_1}$  may be interpreted as follows. This is a two-dimensional entropy density calculated at the scales (or energies) associated with the deformation parameter  $\alpha_2$  by the observer who is at the energies specific for the deformation parameter  $\alpha_1$ .

Contrary to the classical quantum mechanics, this quantity is no longer scalar, but rather a matrix value. The associated matrix seems to be asymmetric

$$S_{\alpha_2}^{\alpha_1} \neq S_{\alpha_1}^{\alpha_2} \quad . \quad (28)$$

The conventional notion of statistical entropy appears in the limit  $\alpha_1 \rightarrow 0$ ,  $\alpha_2 \rightarrow 0$

$$S_0^0 = S = -\text{Tr} \rho \log \rho \quad . \quad (29)$$

In this manner the notion of entropy in the early Universe is more splendid and complicated, including a matrix instead of the number.

In the author's works **Shalyt-Margolin (2004, 1)**, **Shalyt-Margolin (2004, 2)** it has been indicated that the matrix  $S_{\alpha_2}^{\alpha_1}$  may be used for the solution of several problems in a theory of black holes. With this matrix, in particular, one can obtain **Shalyt-Margolin (2004, 1, p.397)**, **Shalyt-Margolin (2004, 2, p.2043)** the well-known Bekenstein – Hawking formula **Bekenstein (1973)** for the entropy of a black hole in the semi-classical approximation

$$S_{BH} = \frac{A}{4\ell_p^2}, \quad (30)$$

where  $A$  - surface area of the event horizon of a black hole. In the works mentioned the author has developed an approach to solve the Hawking's information paradox problem **Hawking (1976)** for black holes using  $S_{\alpha_2}^{\alpha_1}$ .

As demonstrated, when this problem is considered in terms of the introduced matrix (entropy density matrix), there is no information loss at the black hole as, with respect to the infinitely remote observer, the information concerning the initial singularity and black hole singularity is the same **Shalyt-Margolin (2004, 1, Chap. 4)**, **Shalyt-Margolin (2005, pp.72 –74)**

$$S_{1/4}^0 = S_{1/4}^0 \quad . \quad (31)$$

Besides, it is demonstrated that a series expansion of the matrix element  $S_{1/4}^{\alpha}$  in terms of small parameter  $\alpha$  may give quantum corrections for the semi-classical value of the black hole entropy in the right side of formula (29) **Shalyt-Margolin (2006)**.

Nevertheless, note that in the above-mentioned work **Shalyt-Margolin (2006)** the calculation of quantum corrections with the use of the deformed density matrix has been contemplated rather than developed. By the present time, the approach intended to study thermodynamics of black holes, and entropy in particular, using the GUP has been better developed. In an earlier work **Medved, Vagenas, (2004)** GUP has been used to obtain an exact value of the logarithmic correction for entropy of a black hole ; and also the calculation of higher-order corrections has been planned. Based on GUP, in a later work **Bolen, Cavaglia, (2005)** the black hole thermodynamics has been studied in de Sitter and Anti-de Sitter spaces. A group of authors **Cardoso, Berti, Cavaglia, (2005)** has reviewed different methods to estimate the total gravitational energy emitted in higher-dimensional scenarios allowing for the formation of mini-black holes from TeV-scale particle collisions. Of course, GUP should play

an important part in such processes. Finally, it has been shown **Adler, Chen, Santiago, (2001), Chen, Adler (2003) and Chen (2003)** that, owing to GUP, a black hole is evaporated incompletely, having the stable remainder with a mass on the order of the Planck mass. In this work it is suggested that the remainders might form the basis for Dark Matter; considering the validity of GUP, the problems of correcting the black hole temperature are treated.

The notion of symmetry in the deformed theories due to the introduction of new parameters becomes wider. In his work the author **Shalyt-Margolin (2004, 1, Chap. 3)** is concerned with the unitary symmetry problem in the early Universe using the parameter  $\alpha$ . This symmetry group on passage to the limit for  $\alpha \rightarrow 0$  should produce the infinite unitary group  $U$  of quantum mechanics and quantum field theory. The quantum field theory is mentioned here, as in some papers **Shalyt-Margolin (2004, 3), Shalyt-Margolin (2005, 2)** the author has demonstrated that from the deformed quantum mechanics one can proceed to the deformed quantum field theory. Certainly, at such symmetries the density matrix of entropy  $S_{\alpha_2}^{\alpha_1}$  should be retained.

Note that the approach to a quantum theory of the early Universe with the use of the Heisenberg's algebra deformation necessitates extension of the symmetry notion. Specifically, this approach involves the quantum groups **Maggiore (1994)** which also represent the deformed algebraic objects, not the groups in the sense of the standard definition **Weyl (1931)**.

## 6 New concepts in fundamental physics. Holographic principle.

Perfectly new concepts have appeared in fundamental physics in the last fifteen years as regards the quantity of information and entropy contained in cosmological objects and in the Universe as a whole. The quintessence of these concepts is the Hooft-Susskind Holographic Principle that may be formulated in its simplest form as follows **Bousso (2002, Chap.3, Sec. C)**:

“The region  $V$  with boundary of  $A$  is fully described by no more than  $A/4\ell p^2$  degrees of freedom, or about 1 bit of information per Plank area. A fundamental theory, unlike local field theory, should incorporate this counterintuitive result”.



This means that entropy of the region  $V$  satisfies the inequality

$$S(V) \leq \frac{A}{4\ell_p^2}. \quad (32)$$

The Holographic principle provides an answer for the question: “How many degrees of freedom are there in nature, at the most fundamental level?” The Principle has been first put forward in the works of ’t Hooft (1993), (2000) and Susskind (1995).

It should be noted that, as the foregoing formulation presents the principle and not a physical law, we can consider only the objects meeting this principle and the conditions for which this principle is valid.

Initially, the Holographic Principle has been substantiated **Bousso (2002, Chap.2, Sec. C1, Susskind process)**: “Let us consider an isolated matter system of mass  $E$  and entropy  $S$  residing in the space-time  $M$ . It is assumed that the asymptotic structure of  $M$  permits the formation of a black hole. For example,  $M$  is asymptotically flat. And let  $A$  be the area of the circumscribing sphere, i.e., the smallest sphere that fits around the system. However  $A$  is well-defined only if the metric near the system is at least approximately spherically symmetric. This will be the case for all spherically symmetric systems, and for all weakly gravitating systems, but not for strongly gravitating systems lacking spherical symmetry. Besides, we assume that the matter system is stable on a timescale much greater than  $A^{1/2}$ . It persists and does not expand or collapse rapidly, so that the time-dependence of  $A$  will be negligible.”

Then, in accordance with the generalized second law, for black holes we have

**Bekenstein (1974)**

$$S_{matter} \leq S_{BH} = \frac{A}{4\ell_p^2}, \quad (33)$$

where  $S_{BH}$  - entropy of a black hole with the submerged isolated matter system whose surface area of the event horizon equals  $A$ . As has been already mentioned,  $A$  is well-defined only if the metric near the system is at least approximately spherically symmetric.

In the nineties of the last century the Holographic Principle has been proved for a considerably wider class of geometries **Bousso (2002, Chap. 5, 6)**.

The Holographic Principle necessitates radical reconsideration of the existing viewpoints in fundamental physics regarding the degrees of freedom of some system, because the number of degrees of freedom  $N$  calculated from the local quantum field theory for the system within the volume  $V$  is proportional to  $V$  **Bousso (2002, Chap. 3, Sec. C)**

$$N \sim V. \quad (34)$$

Actually, the Holographic Principle is an important step in the direction of a unified theory embracing gravity and quantum field theory. Moreover, the Principle leads to the effective dimensionality reduction due to the fact that by this principle all information about the object is concentrated on the surface and hence its effective dimensionality is less by one. Specifically, the 3D stationary objects meeting the Holographic Principle are virtually specified by their 2D boundary. In the general theory similar situation is observed with a random number of measurements: a physical system given in the volume  $V$  at the  $n$ -metric manifold  $M$  and meeting the Holographic Principle is determined by its  $(n - 1)$ -metric boundary  $A$ . Naturally, a symmetry of  $\tilde{G}(V) \rightarrow G(A)$ , where  $\tilde{G}(V)$  is the initial symmetry group of the system within the volume  $V$ , and  $G(A)$  - corresponding symmetry group of the same system projected to the boundary  $A$ .

Finally, it should be noted that the initially formulated Holographic Principle has been recently generalized **Bousso (2002, Chap. 8, Sec. B)** to assume that “... it is a law of physics which must be manifested in the underlying theory. This theory must be a unified quantum theory of matter and space-time”.

## 7 Conclusion

In conclusion, we revert to the stable deformation that has been considered at the beginning. The Heisenberg's algebra deformations are introduced due to the involvement of GUP and minimal length in quantum mechanics. These deformations are stable in the sense of **Definition 2** given in **Section 1**. But this is not true for the unified algebra of Heisenberg and Poincaré. This algebra does not carry the indicated immunity. It is suggested that the Lie algebra for the interface of the gravitational and quantum realms is in its stabilized form. Now it is clear that such a stability should be raised to the status of a physical principle. In a very interesting

work of **Ahluwalia – Khalilova (2005)** it has been demonstrated that the stabilized form of the Poincaré-Heisenberg algebra **Vilela-Mendes (1994), Chryssomalakos, Okon (2004)** carries three additional parameters: “a length scale pertaining to the Planck/unification scale, a second length scale associated with cosmos, and a new dimensionless constant with the immediate implication that ‘point particle’ ceases to be a viable physical notion. It must be replaced by objects which carry a well-defined, representation space dependent, minimal spatiotemporal extent”.

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